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Research papers

Theoretical and numerical analysis of vertical distribution of active particles in a free-surface wetland flow

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A R T I C L E I N F O

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A B S T R A C T

Presented in this paper is a theoretical and numerical analysis for active particles in a fully developed steady wetland flow dominated by the free-surface effect. An ecological risk assessment model for the concentration distribution of active particles is devised as an extension of the general form of concentration transport equation for passive particles in wetland flows. The vorticity in the free-surface wetland flow is found to depend on the dimensionless parameter α, which reflects the combined action of vertical momentum dispersion, microscopic curvature of flow passages, friction of vegetation and water depth. The large α results in the decrease of vorticity at the free surface and the increase at the bed bottom. The analytical solution of stable concentration distribution is rigorously derived for the active particles in both weak and strong vortical flows, under the combined action of the effective mass dispersion by the ambient flow, as well as the translational diffusion and vertical swimming by the active particles. It is found that the strong vorticity weakens the concentration of active particles in the free-surface wetland flow, while the strong diffusion, by the wetland flow and active particles, enhances the concentration. The large α results in the increase of concentration near the free surface and the decrease of concentration near the bed bottom. The time scale for active particles to reach the stable concentration distribution is mainly dependent on the dimensionless parameter, *Pe*, which reflects the relative strength of the vertical swimming and the total diffusion due to the wetland flow and the active particles.

1. Introduction

Wetlands have significant ecosystem service values, in terms of water purification, groundwater recharge, growth of living materials, protection of biodiversity (Mitsch and Gosselink, 1993; Costanza et al., 1997; Noor Islam et al., 2014; Mujere and Eslamian, 2015), etc. For ecological risk assessments and ecological restoration associated with wetlands, an essential issue is to determine the vertical distribution of particles wherein, including the active and passive particles (Nepf et al., 2007; Chen et al., 2010; Nepf, 2012; Luo et al., 2016; Zeng and Pedley, 2018).

There are a large number of passive particles in wetlands, such as mineral particles, soil particles, plastic pellets, etc. The passive particles, in general, move by means of advection caused by the ambient flow, Brownian motion due to random collisions, as well as the density difference of them and the ambient fluid. Regarding the concentration distribution of passive particles in wetland flows, many efforts have been made, with focus on the dispersion of soluble materials under physical, chemical, and biological processes, etc. There are three typical methods for analyzing the solute dispersion and concentration, including Taylor's analysis on dispersion (Taylor, 1953; Taylor, 1954), method of concentration moments (Aris, 1956, 1960), and method of multi-scale expansion (Mei et al., 1996).

The physical processes related to the distribution of passive particles mainly depend on the hydrodynamic conditions, the morphology and distribution of vegetation, wind stress exerted on the free water surface, etc. Lightbody and Nepf (2006a,b) presented a formula of longitudinal dispersion coefficient for the soluble materials in the flow through a salt marsh fully vegetated by emergent vegetation. Murphy et al. (2007) and Nepf and Ghisalberti (2008) analyzed the behaviors of longitudinal dispersion in turbulent flow. Based on the method of concentration moments and the method of multi-scale expansion, the transport and distribution of passive particles have been analyzed for a single-zone wetland flow (Zeng and Chen, 2011; Zeng et al., 2015; Wu et al., 2011b,c, 2012; Chen, 2013), as well as the two- and three-zone (layer) wetland flows (Chen et al., 2011; Wang et al., 2013, 2014; Wu et al.,

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Nomenclature

B	gyrotactic parameter to reflect the time scale for reorientation of active particle, s.
C	concentration of active particles, kg m^{-3}.
C_0	concentration of active particles at the bed bottom, kg m^{-3}.
D_{cw}	parameter to reflect the relative strength of the effective mass dispersion in the vertical direction due to the wetland flow and the vertical translational diffusion due to particles' swimming.
D_r	rotary diffusivity, s^{-1}.
K_{ij}	component of mass dispersivity tensor caused by the wetland flow, m^2 s^{-1}.
K_V	vertical mass dispersivity, m^2 s^{-1}.
H	water depth, m.
Pe	parameter to reflect the relative strength of the mean swimming and the total diffusion, dimensionless.
Pe_s	parameter to reflect the relative importance of active particles' swimming and vertical translational diffusion, dimensionless.
t	time, s.
t_0	time scale for concentration distribution to reach the stable status, s.
u_i	velocity component of ambient flow, m s^{-1}.
u_i^s	velocity component of active particles swimming, m s^{-1}.
$\boldsymbol{u}^s$	swimming velocity in the weak vortical flow, m s^{-1}.
$u(\zeta)$	velocity at the vertical position ζ, m s^{-1}.
U_m	depth-averaged velocity, m s^{-1}.
V_s	swimming speed of active particles m s^{-1}.
w^s	vertical swimming velocity in the weak vortical flow, m s^{-1}.
x_i, x_j	spatial coordinates, m.
α	dimensionless parameter to represent the combined action of vertical momentum dispersion, microscopic curvature of flow passage, friction of vegetation, and water depth, dimensionless.
ζ	dimensionless vertical coordinate, dimensionless.
θ	polar angle in the spherical coordinated system, dimensionless.
κ	tortuosity, dimensionless.
λ	mass diffusivity, m^2 s^{-1}.
λ_{ij}^s	component of translational diffusivity tensor, m^2 s^{-1}.
λ_{xx}^s, λ_{yy}^s, λ_{zz}^s	diagonal component of translational diffusivity tensor, m^2 s^{-1}.
σ	parameter to represent the relative strength of cells' reorientation and rotational diffusion, dimensionless.
τ	dimensionless time, dimensionless.
ϕ	porosity, dimensionless.
ψ_0	dimensionless vertical swimming velocity in the weak vortical flow, dimensionless.
ψ_V^r	dimensionless vertical swimming velocity, dimensionless.
$\boldsymbol{\omega}$	vorticity, (m^{-1}).
ω_0	parameter to represent the relative strength of flow shear to reorientation of active particles, dimensionless.
ω_r	dimensionless vorticity, dimensionless.
ω_r^{max}	maximum vorticity, dimensionless.
ω_r^{min}	minimum vorticity, dimensionless.
ω_x, ω_y	horizontal components of vorticity, m^{-1}.
Ω	dimensionless time, dimensionless.
g_1'	differentiation of g_1 with respect to $\cos(\theta)$.
∇	the del operator.

2011a, 2015, 2016; Luo et al., 2016, 2017). Wind also plays an important role in determining the vertical distribution of passive particles in the wetland flow, by exerting a shear stress on the water surface. Some ecological indicators were also presented to evaluate the effects of wind on contaminant transport (Guo et al., 2018).

Chemical and biological processes are also important for the distribution of passive particles (US EPA, 1999; Sağ et al., 2000; Fereidouni et al., 2009; Chiban et al., 2011). By use of an exponential transformation, Zeng and Chen (2011) presented an analytical solution for the evolution of depth-averaged concentration of single component contaminant due to an irreversible reaction and hydraulic dispersion. Recently, some analytical endeavors have been made to reveal the characteristic of bicomponent contaminant transport in wetland flows dominated by the free-surface and bank-wall effects (Chen et al., 2012; Zeng et al., 2014).

The distribution of active particles in wetland flows is more complex than that of passive particles, due to the swimming behaviors of active particles (Pedley and Kessler, 1990, 1992; Hill and Bees, 2002; Bearon et al., 2011; Goldstein, 2015). Regarding the concentration distribution of active particles, there have been various theoretical, experimental and numerical investigations, with focus on the distribution of gyrotactic micro-organisms in typical flows in the absence of vegetation, such as the flow through a vertical tube or channel (Kessler, 1985, 1986; Hwang and Pedley, 2014), the horizontal shear flow between two parallel walls (Durham et al., 2009, 2013), the flow in a rotating cylindrical vessel (Lillo et al., 2014), the density stratified flow (Ardekani et al., 2017), the free surface flow (Lovecchio et al., 2014, 2017; Mashayekhpour et al., 2017; Enriquez and Taylor, 2015), and the turbulent channel flow in photobioreactors (Croze et al., 2013). However, the distribution of active particles in the wetland flow have not been understood very well. Very recently, Zeng and Pedley (2018) investigated the concentration distribution of gyrotactic micro-organisms in the flow past a single vertical circular cylinder, which is a simple proxy of the flow through the wetland with sparse vegetation. A reliable ecological risk assessment and ecological restoration in the wetland flow requires a reasonable understanding of the behavior of both active and passive particles. However, the current analytical efforts mainly focus on the distribution of passive particles, and up to now, no analytical solution has been presented to predict the concentration distribution of active particles in the free-surface wetland flow.

This work is to investigate the concentration distribution of active particles in the free-surface wetland flow. The active particles considered here are gyrotactic micro-organisms, which widely exist in wetlands. Many significant ecological phenomena are caused by gyrotactic micro-organisms. For example, the red tide in the coastal region is often caused by the massive growth and accumulation of *Heterosigma akashiwo*. The specific objects are: (I) to formulate the typical case of concentration evolution of active particles in the free-surface wetland flow, (II) to obtain the analytical solution of vorticity in the free-surface wetland flow, (III) to find the concrete expression for stable distribution of active particles in both weak and strong vortical flows, and (IV) to determine the effects of typical parameters on the stable concentration distribution and the time scale to reach the stable status.

2. Formulation

In the present work, the term, active particles, refers to the gyrotactic micro-organisms which execute directional or random motions by rotating, waving or undulating flagella, while the term, passive particles, refers to the non-living particles without the ability to convert its internal energy to kinetic motion, for example, mineral particles. The governing equation for the concentration transport of active particles in the wetland flow can be adopted at the phase average scale as (Liu et al., 2005; Chen et al., 2010; Chen, 2013)

$$\phi\frac{\partial C}{\partial t} + \frac{\partial}{\partial x_i}[(u_i + u_i^s)C] = \frac{\partial}{\partial x_i}\left(\kappa\lambda\phi\frac{\partial C}{\partial x_i}\right) + \kappa\frac{\partial}{\partial x_i}\left[(K_{ij} + \phi\lambda_{ij}^s)\frac{\partial C}{\partial x_j}\right] \tag{1}$$

where ϕ (dimensionless) is porosity, C (kg m^{-3}) the concentration of active particles, t (s) time, x_i (m) and x_j (m) the spatial coordinates ($i, j = 1, 2, 3$), u_i (m s^{-1}) the velocity component of ambient flow, u_i^s (m s^{-1}) the velocity component of active particles swimming, κ (dimensionless) the tortuosity, λ (m^2 s^{-1}) the mass diffusivity, K_{ij} (m^2 s^{-1}) the component of mass dispersivity tensor caused by the wetland flow, and λ_{ij}^s (m^2 s^{-1}) the component of translational diffusivity tensor caused by active particles. The diffusion term K_{ij} exists for both passive and active particles. The term λ_{ij}^s is related to the rate of strain, vorticity, as well as cells' morphology. In general, the term λ_{ij}^s varies with position for a velocity profile with spatially varying rate of strain and vorticity. However, for the spherical gyrotactic cells, λ_{ij}^s is only dependent on the vorticity. As the first step of theoretical analysis of the distribution of gyrotactic micro-organisms in the wetland flow, the micro-organisms considered in the present work are approximately spherical micro-organisms, such as *Chlamydomonas*, *Dunaliella*, etc., which are found in many important ecological phenomena associated with wetland flows, for example harmful algal blooms. Furthermore, the contribution of λ_{ij}^s to the concentration distribution of micro-organisms, is mainly related to its diagonal components rather than its off-diagonal components, since the latter are much less than the former. Also, the existence of horizontal vorticity can weaken the difference between the horizontal components, λ_{xx}^s and λ_{yy}^s, and vertical component, λ_{zz}^s. For the case of large ambient vorticity, $\lambda_{xx}^s \approx \lambda_{yy}^s \approx \lambda_{zz}^s$. Even for the case of vorticity equal to zero, λ_{zz}^s is still comparable to the horizontal components. It is noted that the strong vorticity can change the mean swimming velocity greatly, resulting in the large variation of concentration distribution of micro-organisms. Many active particles, say *Chlamydomonas reinhardtii*, exhibit the behavior of diel vertical migration (diurnal vertical migration) that the active particles swim upwards for photosynthesis in the daytime and settle down to gain nutrients at light.

Consider a fully developed steady free-surface wetland flow with constant ϕ, κ, K_{ij} and λ_{zz}^s, in the Cartesian coordinate system with the longitudinal x-axis aligned with the flow direction, the vertical z-axis upwards, and the origin at the bottom bed, as shown in Fig. 1. Consider the concentration distribution of active particles with the initial concentration zero in the free-surface wetland flow, caused by the upward migration of the active particles from the bottom layer with constant concentration C_0. The concentration distribution is uniform in the streamwise direction, and the governing equation Eq. (1) reduces to

$$\frac{\partial C}{\partial t} + \frac{1}{\phi}\frac{\partial(w^s C)}{\partial z} = \kappa\left(\lambda + \lambda_{zz}^s + \frac{K_V}{\phi}\right)\frac{\partial^2 C}{\partial z^2} \tag{2}$$

where w^s (m s^{-1}) is the vertical swimming velocity of active particles, and K_V (m^2 s^{-1}) is the vertical mass dispersivity.

For the free water surface, we have zero-flux boundary condition

$$\left[\frac{w^s C}{\phi} - \kappa\left(\lambda + \lambda_{zz}^s + \frac{K_V}{\phi}\right)\frac{\partial C}{\partial z}\right]\Bigg|_{z=H} = 0 \tag{3}$$

where H (m) is the water depth, which means that the active particles cannot escape from the water body through the free surface.

With dimensionless parameters of

$$\Omega = \frac{C}{C_0}, \quad \tau = \frac{H^2}{\kappa\left(\lambda + \lambda_{zz}^s + \frac{K_V}{\phi}\right)}, \quad \zeta = \frac{z}{H}, \quad \psi_V^r = \frac{w^s}{V_s} \tag{4}$$

where V_s (m s^{-1}) denotes the swimming speed of active particles, the governing equation, boundary conditions and initial conditions for the concentration distribution can be rewritten as

$$\begin{cases} \frac{\partial\Omega}{\partial\tau} + \frac{Pe^s}{(1 + D_{cw})\phi\kappa}\left(\psi_V^r\frac{\partial\Omega}{\partial\zeta} + \Omega\frac{\partial\psi_V^r}{\partial\zeta}\right) = \frac{\partial^2\Omega}{\partial\zeta^2} \\ \Omega|_{\zeta=0} = 1, \quad \left[\frac{Pe^s}{(1 + D_{cw})\phi\kappa}\psi_V^r\Omega - \frac{\partial\Omega}{\partial\zeta}\right]\Bigg|_{\zeta=1} = 0 \\ \Omega(\zeta, 0) = 0 \end{cases} \tag{5}$$

with

$$Pe^s = \frac{V_s H}{\lambda_{zz}^s}, \quad D_{cw} = \frac{(\lambda + K_V/\phi)}{\lambda_{zz}^s} \tag{6}$$

where Pe^s (dimensionless) reflects the relative importance of active particles' swimming and vertical translational diffusion, and D_{cw} (dimensionless) reflects the relative strength of the effective mass dispersion in the vertical direction due to the wetland flow and the vertical translational diffusion due to particles' swimming.

3. Vorticity distribution in the free-surface wetland flow

For the swimming of gyrotactic micro-organisms, vorticity ω (m^{-1}) is an important indicator to influence the swimming velocity and direction, especially for the spherical gyrotactic micro-organisms (Pedley and Kessler, 1992). For the weak vorticity field, the active particles can swim upwards in the non-vertical direction, while, for the strong voriticity filed, they tumble unsteadily. Vorticity is a pseudo vector field to reflect the local spinning motion of a continuum, defined as $\boldsymbol{\omega} = \nabla \times \boldsymbol{u}$, where ∇ is the del operator.

For the free surface wetland flow, the vertical velocity profile has been given by Zeng and Chen (2011)

$$u(\zeta) = U_m\frac{\cosh\alpha - \cosh[\alpha(1 - \zeta)]}{(\alpha\cosh\alpha - \sinh\alpha)/\alpha} \tag{7}$$

where U_m (m s^{-1}) is the depth-averaged velocity, and α is a dimensionless parameter to represent the combined action of vertical momentum dispersion, microscopic curvature of flow passage, friction of vegetation, as well as water depth. Therefore, the dimensionless vorticity, ω_r, for the free-surface wetland flow can be expressed as

$$\omega_r = \frac{\omega}{1/B} = \omega_0\frac{\alpha^2\sinh[\alpha(1 - \zeta)]}{\alpha\cosh\alpha - \sinh\alpha} \tag{8}$$

where B (s) is the gyrotactic parameter to reflect the time scale for reorientation of active particles by the gravitational torque against the viscous torque exerted by the shear of ambient flow, and $\omega_0 = BU_m/H$ reflects the relative strength of the flow shear and reorientation of active particles. Fig. 2(a) presents the variation of ω_r with ζ for various α. The maximum vorticity $\omega_r^{max} = \omega_0\alpha^2\sinh(\alpha)/(\alpha\cosh\alpha - \sinh\alpha)$ always appears at the bed bottom, while the minimum vorticity $\omega_r^{min} = 0$ always exists at the free water surface. In contrast to the dependence of the maximum vorticity on α, the minimum vorticity keeps constant. ω_r increases with the increase of α for the region near the free water surface, while it decreases for the region near the bottom. Fig. 2(b) presents the variation of ω_r with ζ for various ω_0. It is shown that the larege ω_0 can enhance the vorticity in the whole region.

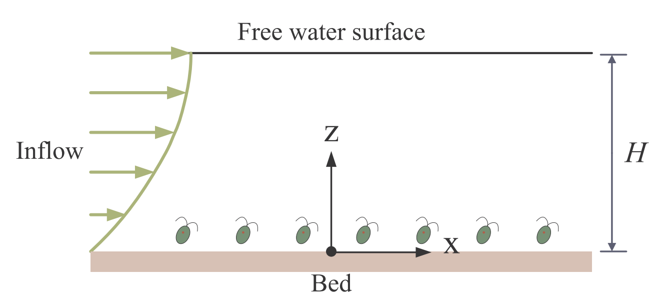

Fig. 1. Sketch for a free-surface wetland flow.

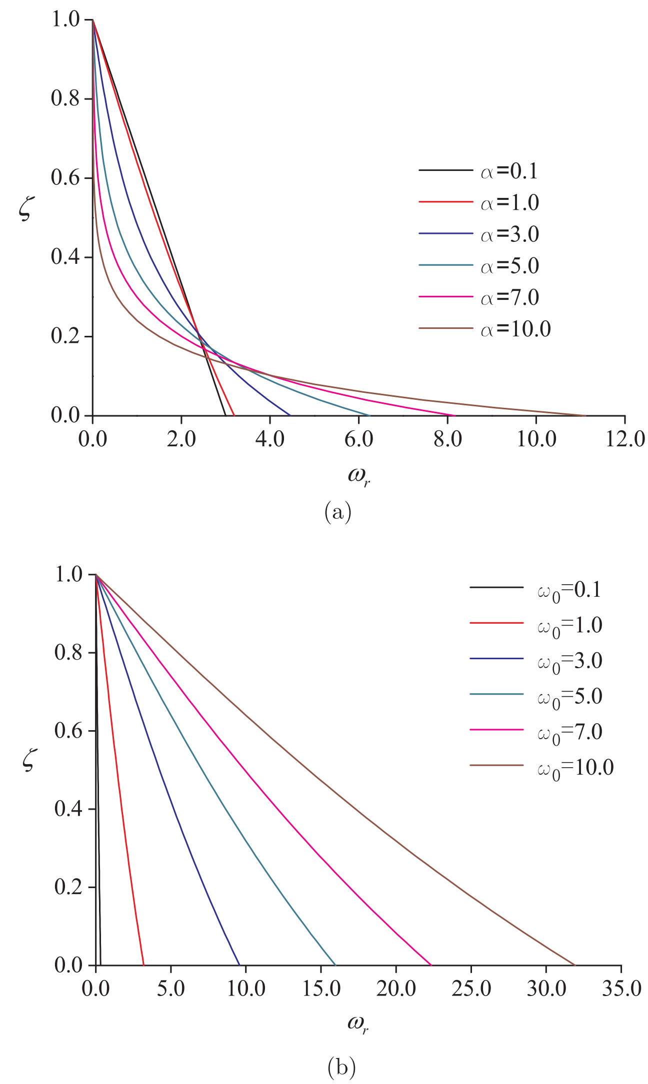

Fig. 2. Variation of ω_r with ζ: (a) $\omega_0 = 1.0$, and (b) $\alpha = 1.0$.

4. Stable distribution of active particles in the weak vortical flow

For the long time evolution of passive particles, the concentration distribution can reach a asymptotic stable status. Further more, the swimming velocity of gyrotactic micro-organisms for the weak vorticity flow of $B|\boldsymbol{\omega}| \ll 1$ can be expressed, in the present notation, as (Pedley and Kessler, 1990)

$$\frac{\boldsymbol{u}^s}{V_s} = (B\omega_y J_1, -B\omega_x J_1, \psi_0) \tag{9}$$

where ω_x (m^{-1}) and ω_y (m^{-1}) are the horizontal components of $\boldsymbol{\omega}$, and J_1 (dimensionless) and ψ_0 (dimensionless) are the parameters associated with B and D_r(m^2 s^{-1}), where D_r is the rotary diffusivity to reflect the randomness of active particles. ψ_0 can be expressed as (Pedley and Kessler, 1990)

$$\psi_0 = \coth(\lambda) - \frac{1}{\sigma}, \quad \sigma = \frac{1}{BD_r} \tag{10}$$

where σ represents the relative strength of cells' reorientation and rotary diffusion. The large σ means the relative strong gyrotaxis, while the small σ is corresponding to the strong randomness. J_1 can be expressed, in the present notation, as (Pedley and Kessler, 1990)

$$J_1 = \pi\frac{\sigma^2}{4\pi\sigma\sinh(\sigma)}\int_{-1}^{1}(1-\eta^2)^{\frac{1}{2}}g_1(\eta)d\eta \tag{11}$$

where η is the integral variable, and $g_1(\eta)$ is subject to

$$\frac{d}{dq}[(1-\eta^2)g_1'] - \frac{g_1}{1-\eta^2} - \sigma\frac{d}{dq}[(1-\eta^2)g_1] = -\sigma(1-\eta^2)^{\frac{1}{2}}\exp(\sigma\eta) \tag{12}$$

where g_1' denotes for the differentiation of g_1 with respect to $q = \cos(\theta)$, with θ standing for the polar angle in the spherical coordinate system in which $\theta = 0$ is vertically upward.

Therefore, for the case of weak vortical flow of

$$\omega_0\frac{\alpha^2\sinh[\alpha(1-\zeta)]}{\alpha\cosh\alpha - \sinh\alpha} \ll 1 \tag{13}$$

the vertical swimming velocity is independent of position, and the definite problem described by Eq. (5) can be simplified as

$$\begin{cases} \frac{Pe^s}{(1+D_{cw})\phi\kappa}\psi_0\frac{d\Omega}{d\zeta} = \frac{d^2\Omega}{d\zeta^2} \\ \Omega|_{\zeta=0} = 1, \quad \left[\frac{Pe^s}{(1+D_{cw})\phi\kappa}\psi_0\Omega - \frac{d\Omega}{d\zeta}\right]\Big|_{\zeta=1} = 0 \end{cases} \tag{14}$$

Eq. (14) is a second-order linear ordinary differential equation with constant coefficients, and its solution is

$$\Omega(\zeta) = \exp(Pe\psi_0\zeta) \tag{15}$$

where $Pe = Pe^s/[(1 + D_{cw})\phi\kappa]$ reflects the relative strength of the mean swimming and the total diffusion, including the effective mass dispersion in the vertical direction caused by the wetland flow and the vertical translational diffusion caused by the active particles. The concentration distribution of active particles is subject to the exponential distribution in the weak shear flow. The concentration at the free surface increases with the increase of upward swimming velocity, because the large vertical swimming velocity means the large Pe_s for a given wetland flow. For the case of positive ψ_0, the concentration of active particles increases with ζ. Although active particles swim upward continuously, the concentration at the free surface cannot increase indefinitely due to the diffusive effects by the active particles and the wetland flow. For the case of negative ψ_0, which represents the settling of active particles, the concentration decreases with the increasing ζ. For the limiting case of $\psi_0 = 0$, Eq. (19) reduces to $\Omega(\zeta) = 1$, which implies that the concentration of non-motile particles eventually reach uniform distribution in the vertical direction.

5. Stable distribution of active particles in the strong vortical flow

For the case of strong vortical flow, $\boldsymbol{\omega}$ does not satisfy the condition of $B|\boldsymbol{\omega}| \ll 1$, and the effect of horizontal vorticity on the vertical swimming velocity cannot be neglected. In general, the vertical swimming velocity varies with ζ for the strong vortical flow, and the definite problem described by Eq. (5) reduces to

$$\begin{cases} \frac{Pe^s}{(1+D_{cw})\phi\kappa}\left(\psi_V^r\frac{d\Omega}{d\zeta} + \Omega\frac{d\psi_V^r}{d\zeta}\right) = \frac{d^2\Omega}{d\zeta^2} \\ \Omega|_{\zeta=0} = 1, \quad \left[\frac{Pe^s}{(1+D_{cw})\phi\kappa}\psi_V^r\Omega - \frac{d\Omega}{d\zeta}\right]\Big|_{\zeta=1} = 0 \end{cases} \tag{16}$$

In contrast to the concentration distribution of active particles in the weak vortical flow in which the vertical swimming influences the concentration distribution only by the vertical variation of cells' concentration, an additional term (source term) $\Omega\frac{d\psi_V^r}{d\zeta}$ appears for the strong vortical flow due to the variation of vertical swimming velocity.

Solving Eq. (16) gives

$$\Omega(\zeta) = \frac{c_1\int_1^0\exp(\int_1^0 -Pe\psi_V^r d\zeta)d\zeta + c_2}{\exp(\int_1^\zeta -Pe\psi_V^r d\zeta)} \tag{17}$$

where c_1 and c_2 are the undetermined constants related to the concentration boundary conditions at the free surface and bed bottom.

Substituting the concentration boundary conditions in Eq. (16) into

Eq. (17) yields

$$c_1 = 0, \quad c_2 = \exp\left(\int_1^0 -Pe\psi_V^r \mathrm{d}\zeta\right) \tag{18}$$

Therefore, the concentration distribution of active particles in the strong vertical flow can be expressed as

$$\Omega(\zeta) = \exp\left(\int_0^\zeta Pe\psi_V^r \mathrm{d}\zeta\right) \tag{19}$$

which means that the concentration of active particles at ζ is independent of swimming in the region above ζ.

To illustrate the effects of Pe, α, and ω_0 on the concentration distribution of active particles, *Chlamydomonas reinhardtii* has been chosen in the present work. The swimming velocity of *Chlamydomonas reinhardtii* can be expressed, in the present notation, as (Liu, 2018)

$$\psi_V^r = \begin{cases} a_0 + \frac{a_1\exp(a_3 - a_4\omega_r)}{[1+\exp(a_2 - a_4\omega_r)][1+\exp(a_3 - a_4\omega_r)]}, & 0 \leqslant \omega_r < b_0 \\ a_5\omega_r^{a_6}, & b_0 \leqslant \omega_r \leqslant b_1 \end{cases} \tag{20}$$

where $a_0 = 4.700 \times 10^{-2}$, $a_1 = 9.800 \times 10^{-1}$, $a_2 = -2.807 \times 10^{-6}$, $a_3 = 2.577 \times 10^{-6}$, $a_4 = -8.936 \times 10^{-1}$, $a_5 = 9.255 \times 10^{-1}$, $a_6 = -1.829$, $b_0 = 4.089$, and $b_1 = 5.918 \times 10^1$.

In general, the size of a gyrotactic micro-organism is much less than that of stem spacing in a typical free-surface wetland. The mean diameter of most gyrotactic mirco-organisms lies in the range of 1 μm–200 μm, while, in general, the size of the stem spacing lies on the order of 1 cm and 10 cm for the free-surface wetland. Therefore, the expression of swimming velocity, Eq. (20), for micro-organisms in free space can be adopted. However, it may be not a reasonable choice for porous media with the size of pores comparable to the size of a micro-organism.

For the case of $\omega_r > b_1$, the vertical swimming velocity tends to approach constant zero. Similar results have been reported for *Heterosigma akashiwo* in a three-dimensional vorticity field (Chen et al., 2018). The reason is that the strong horizontal vorticity causes micro-organisms to tumble. Some important ecological phenomena (for example, formation of thin phytoplankton layer in oceans) can be triggered by the strong horizontal vorticity (Durham et al., 2009).

Fig. 3 presents the variation of Ω with ζ for various α, ω_0 and Pe. Ω increases with the increase of α near the free surface, while it decreases with the increasing α near the bed bottom, as shown in Fig. 3(a). The strong ambient vorticity results in a small concentration, as shown in Fig. 3(b), since the strong vorticity weakens the vertical swimming velocity of active particles. Ω increases with the increase of Pe, as shown in Fig. 3(c), because the large Pe represents a strong effective vertical diffusion for a given swimming velocity.

6. Time scale for active particles to reach the asymptotic stable distribution

For the time evolution of active particles during the initial stage, Ω varies with the vertical position ζ and time τ. Here, the finite difference method is employed to compute the concentration distribution.

Fig. 4 presents the variation of Ω with τ for $\zeta = 0.5$: (a) $Pe = 1.0$ and $\omega_0 = 1.0$, (b) $\alpha = 1.0$ and $Pe = 1.0$, and (c) $\alpha = 1.0$ and $\omega_0 = 1.0$. It is shown that the concentration of active particles gradually increases to reach a stable status under the combined action of vertical swimming, the translational diffusion by the active particles, as well as the effective mass dispersion by the wetland flow, which is similar to the asymptotic

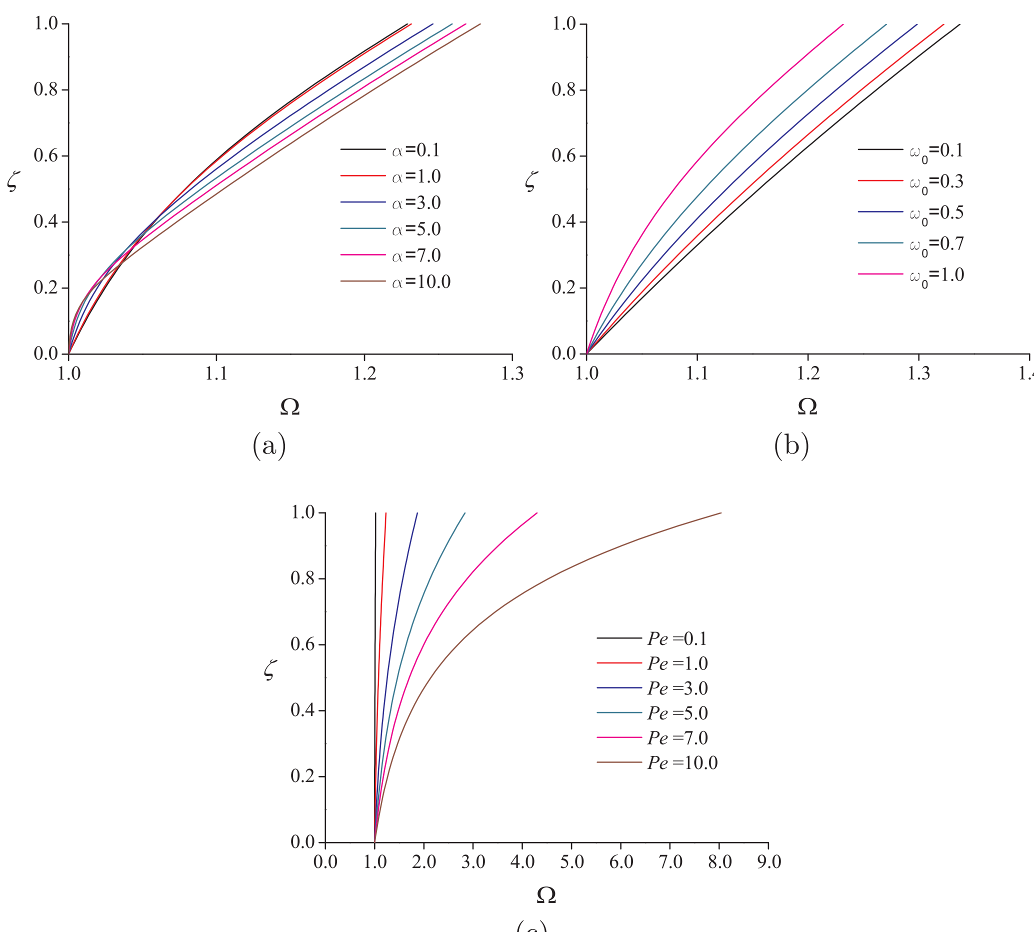

Fig. 3. Variation of Ω with ζ: (a) $Pe = 1.0$ and $\omega_0 = 1.0$, (b) $\alpha = 1.0$ and $Pe = 1.0$, and (c) $\omega_0 = 1.0$ and $\alpha = 1.0$.

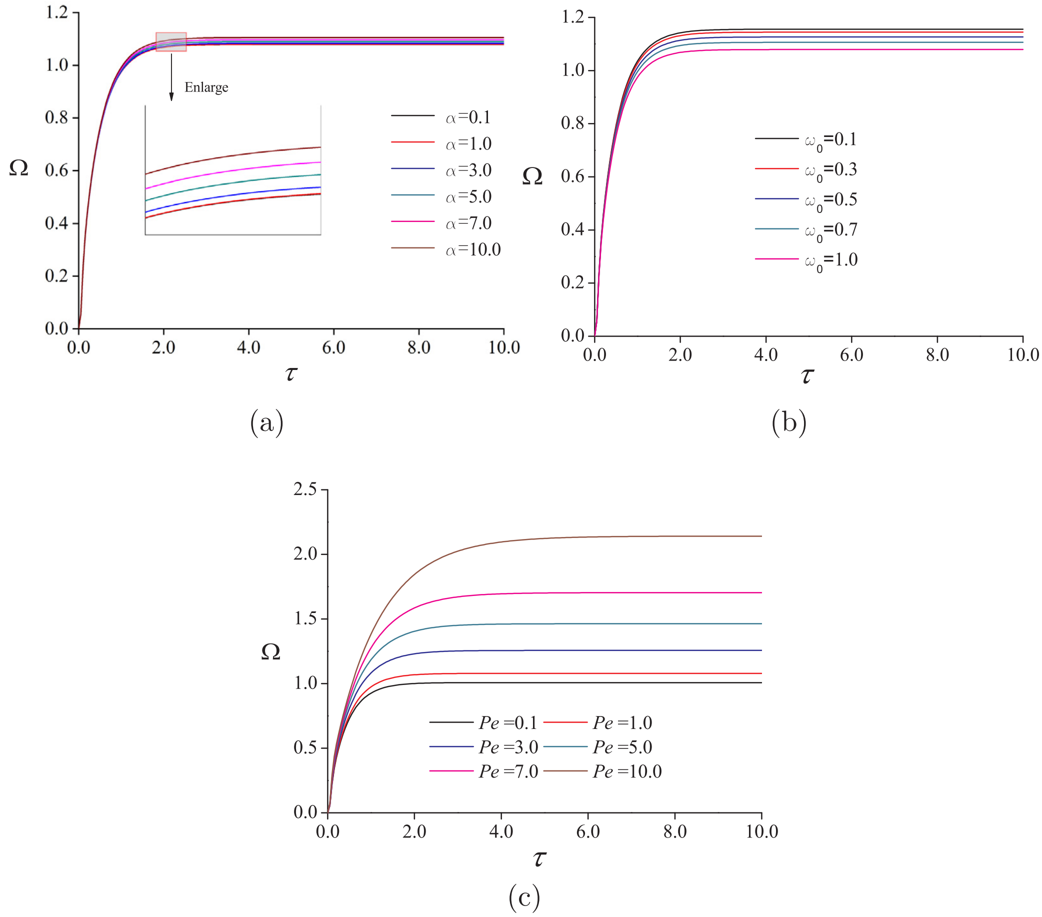

Fig. 4. Variation of Ω with τ for $\zeta = 0.5$: (a)$Pe = 1.0$ and $\omega_0 = 1.0$, (b) $\alpha = 1.0$ and $Pe = 1.0$, and (c) $\omega_0 = 1.0$ and $\alpha = 1.0$.

characteristics of concentration transport of passive particles in the free-surface wetland flow (Chen et al., 2010; Zeng and Chen, 2011).

The time scale t_0 (s) for the concentration distribution to reach the stable status is mainly influenced by *Pe*. t_0 increases with the increase of *Pe*, while it decreases with the increase of ω_0. The reason is that the large *Pe* implies the strong vertical swimming or the weak vertical diffusion. t_0 is not sensitive to the variation of α for the given parameters *Pe* and *B*.

7. Conclusions

With essential implications to ecological risk assessments and restoration associated with wetlands, a typical case of active particles transport in the free-surface wetland flow is concretely formulated by extending the basic equation of concentration transport adopted for passive particles in wetland flows.

It is found that the vorticity for the free-surface wetland flow depends on two dimensionless parameters, α and ω_0. The large α can enhance the vorticity near the free water surface, while it weakens the vorticity near the bed bottom. The vorticity always increases with the increase of ω_0. For the large ω_0, the gyrotactic micro-organisms probably tumble unsteadily.

The solution of concentration distribution of active particles is derived rigorously for the free-surface wetland flow with weak vorticity. Results show that the concentration of active particle is subject to an exponential distribution. The upward swimming results in the special distribution pattern of particles with high concentration at the free surface and low concentration at the bed bottom. For the limiting case of $\psi_0 = 0$, the active particles distribute uniformly through the whole depth, due to the total diffusion by the wetland flow and the active particles. The concentration distribution of passive particles, settling with a constant speed, can be included as a special case of negative ψ_0.

The analytical solution of concentration distribution is also obtained for long time evolution of active particles in the free-surface wetland flow with the strong vorticity. Results show that the concentration distribution of active particles can be expressed in the form of $\exp(\int_0^\zeta Pe\psi_V^r d\zeta)$, which, in general, is not subject to the exponential distribution of ζ as in the weak vortical flow. The vertical concentration depends on three dimensionless parameters *Pe*, α and ω_0. The large *Pe* and small ω_0 can enhance the concentration of active particles through the whole water depth. However, the enhancement of concentration due to the increase of α only occurs near the free surface, which is consistent with the effect of α on the vorticity distribution. Numerical results show that the time scale, t_0, for active particles to reach the stable distribution mainly depends on *Pe*. t_0 increases with the increase of *Pe*.

The analytical solution presented in this work provides one potential way to estimate the concentration distribution of gyrotactic micro-organisms in the free-surface wetland flow, which may be useful and meaningful in the qualitative ecological risk assessments associated with harmful algal blooms. However, the analytical solution of vertical distribution of active particles presented in this work is based on an idealized case in which some simplifications have been made for the flow and micro-organisms. For example, the gyrotactic micro-organisms considered here are the spherical micro-organisms, and the wetland flow considered here is the two-dimensional steady flow. Further work, associated with practical flow conditions, biological characteristics, light, temperature and nutrient levels, should be performed to predict the harmful algal blooms in a realistic wetland flow.

Conflict of interest

None.

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