

## Computational Study of Softrock Erosion in a Mountain River

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### Abstract

JiJi Weir was built across a mountain river, ChoShui River (濁水溪), Taiwan. The flow in the river is strongly dominated by the precipitation of Typhoon in the region. This mountain river carries a large amount of sediment loads in flood seasons. The geology of the site of the JiJi Weir is not strong; the channel immediately downstream is formed by the lithology layers of mud, shiver, and sandstones. Serious scouring of the soft rock bed occurred and the scour hole formed a head-cut migrating upstream toward the Weir. To study the trend of the soft rock scouring and evaluate the headcut migration, the computational model CCHE2D, developed at the National Center for Computational Hydroscience and Engineering, the University of Mississippi, was applied. A computational module for bed rock erosion was developed and tested using the field data. The mechanism of rock erosion due to the impact of sediment particles carried with the flood flows was considered based on a semi-empirical rock abrasion formulation.

### 1. Introduction

Geomorphological changes of landscape on earth surface are caused by many natural mechanisms of erosion due to water flow, wind or weathering. Bedrock abrasion, among others, is one of the main factors to shape the mountain geomorphology. These processes are normally measured with large time scales because these changes are very slow and can hardly be detected by human's eyes. However, when the bed rock is very soft with high erodibility and the flow is very strong and carrying a lot of sediment, the bed rock abrasion process can be quite fast to cause significant channel incision and threaten the integrity of manmade in-stream structures. The soft rock erosion downstream of the Jiji Weir in ChoShui River, Taiwan, is such a problem.

The channel of the ChoShui River immediately downstream of the Jiji Weir, a low dam across the river, is formed by mudstone. According to the report “集集攔河堰下游段河道沖淤與治理策略研究” (Pages 4-6), this Soft Taiwan Rock is of alternating lithology layers of mud, shiver, and sandstones. The tensile strength ranges widely 0.5-25 (MPa). The rock surface often is weakened by weathering. The mudstone layer can be softened

significantly under water. The weakened layers degrade the integrity of the overall base rock and enhance the bed rock scouring rates. This report indicated that some mudstones have swelling and slaking phenomena under wet and dry conditions. The surface of the rock would crack and fall naturally as a result. Some of these weak bed rocks can be eroded significantly by the flow even without considering the effect of moving sediment particle impacts.

Progress in calculating bed rock erosion due to abrasion has been limited. Sklar and Dietrich (1998, 2001 and 2004) developed formulations for predicting bed rock erosion by bed load. Sediment particle saltation and impact on the bed rock were considered to predict the amount of abrasion. Turowski, et al. (2007) improved the Sklar's approach with more detailed covering effect. Stock, et al. (2005) measured rock erosion rate in several field sites including river in the US and Taiwan. The rates of the soft rock erosion in Taiwan were among the highest in the survey.

Based on Lab experiments, Sklar and Dietrich (2001) studied the process of base rock abrasion by moving sediment particles. It was found that

- The erosion rate decreases with the increase of the tensile strength of the base rock.
- The erosion rate increases with the mass of the sediment particles; it will then decrease after a maximum value of the sediment mass rate has been reached. When sediment mass is low, its increase will add more particles to abrade the bed. After the maximum is reached, further increase of sediment will tend to cover the bed and thus reduce saltation and the erosion rate.
- The erosion rate will increase if the sediment particles are harder than the bed rock.

An empirical relation has been proposed later (Sklar and Dietrich, 2004) that the abrasion rate can be expressed as the product of three parameters:

$$E = V_i I_i F_e \quad (1)$$

Where  $V_i$ ,  $I_i$ , and  $F_e$  are the average volume of rock detached per particle impact, the rate of particle impacts per unit area per unit time and the fraction of the bed made up of exposed bed rock, respectively. The expression of  $E$  has been derived from the formulations of these three factors:

$$E = \frac{q_s w_{si}^2 Y}{L_s k_v \sigma_T^2} \left(1 - \frac{q_s}{q_t}\right) \quad (2)$$

$L_s$  is the saltation length given by

$$\frac{L_s}{D_s} = 8 \left(\frac{\tau}{\tau_c} - 1\right)^{0.88} \quad (3)$$

it was modified with a factor to account for the effect that the particle hop length grows rapidly as shear velocity  $u_*$  (m/s) approaches the particle fall velocity  $w_f$  (m/s):

$$\frac{L_s}{D_s} = 8 \left(\frac{\tau}{\tau_c} - 1\right)^{0.88} \left[1 - \left(\frac{u_*}{w_f}\right)^2\right]^{-0.5} \quad \frac{u_*}{w_f} < 1 \quad (4)$$

The saltation length becomes infinite when shear velocity is approaching to sediment fall velocity:  $u_* \rightarrow w_f$ .  $w_{si}$  is the impact velocity of sediment particles on the bed formulated as

$$w_{si} = 2w_{sd} = 0.8 \left[ \left(\frac{\rho_s}{\rho} - 1\right) g D_s \right]^{0.5} \left(\frac{\tau}{\tau_c} - 1\right)^{0.18} \left[1 - \left(\frac{u_*}{w_f}\right)^2\right]^{0.5} \quad \frac{u_*}{w_f} < 1 \quad (5)$$

Where

$w_{si}$  = Impact velocity of sediment particles (m/s)

$L_s$  = Saltation length of sediment particles (m)

$q_s$  = Sediment supply per unit width (kg/m/s)

$q_t$  = Sediment transport capacity (kg/m/s)

$Y$  = Young's modulus of elasticity of base rock (Pa)

$\sigma_T$  = Tensile strength of the rock (Pa)

$k_v$  = Rock strength parameter (-)

$\tau$  = Shear stress over the bed (Pa)

$\tau_c$  = sediment critical entrainment shear stress

$D_s$  = Sediment particle size (m)

$w_{sd}$  = sediment particle descendent velocity (m/s)

$\rho_s, \rho$  = sediment and water density (kg/m<sup>3</sup>)

$g$  = gravitational acceleration (m/s<sup>2</sup>)

The non-dimensional parameter, rock strength parameter,  $k_v$  was found in the order of  $10^{12} - 10^{13}$  according to experimental data (Solar and Dietrich, 2004).

One has to point out that the factor  $\left(1 - \frac{q_s}{q_t}\right)$  and

$\left[1 - \left(\frac{u_*}{w_f}\right)^2\right]$  in the equations (2), (4) and (5) were

introduced intuitively to account for the effects that:

- the bed be covered by sediment particles so saltation impact to the bed is reduced as more bed load is available;
- the saltation length would increase with the flow shear stress to infinity when  $u_* \rightarrow w_f$

The first factor assumes that bed rock erosion stops when the bed load is equal to or higher than its transport capacity; base rock would not be eroded by rolling bed load and sliding sediment particles. The second factor assumes that sediments are totally suspended when  $u_* \geq w_f$  and will never contact the bed during transport. These two assumptions are not fully consistent with the experiment and field observation that even clear water could erode the soft base rock. In addition, in sediment transport research, the criterion,  $u_* \rightarrow w_f$ , is generally considered as the initiation of sediment suspension rather than the complete suspension.

In this study, numerical simulations of bed rock scour using the CCHE2D (Jia, et al., 2002) hydrodynamic and sediment transport model were conducted for achieving the research objectives. The scouring of the bed rock is not a typical sediment transport problem because there are no sediment or bed materials on the rocky bed surface. A new capability has to be developed to handle this

process. The approach of Sklar and Dietrich (2001 and 2004) has been implemented with modification. The numerical model was calibrated and validated using a part of measured channel change data.

With the Equations (4) and (5), Equation (2) can be used to compute the bed rock incision due to sediment particle saltation. The sediment transport capacity can be calculated using one of the available sediment transport formulas, and the local sediment load can be solved by the CCHE2D sediment transport model (Wu, et al., 2000). The site specific parameter  $Y / \sigma_T^2$  is unknown which can be used as a calibration parameter. In general, the value of  $Y / \sigma_T^2$  for the mudstone and soft shale should be in the order of  $\sim 10^{-3}$ . When applied to the downstream scouring problems of the JiJi Weir, this value would be of a reference because the base rock there is of alternating layers. Some of the soft mud rock layer can easily be eroded and washed away by high speed clear water; those rock layers relatively strong would then be broken by the flow with large sediment particles.

## 2. Modification of the Bed Rock Erosion Model

As mentioned above, Sklar and Dietrich (2004) proposed a rock abrasion model based on their experiment data and hydraulics analysis. Substituting equations 4 and 5 into 2, the erosion rate of rock surface by saltating sediment particles can be expressed as

$$E = 0.08 \frac{Y}{k \sigma_T^2} \left[ \left( \frac{\rho}{\rho} - 1 \right) g \right] q_s \left( 1 - \frac{q_s}{q_t} \right) \left( \frac{\tau}{\tau_c} - 1 \right)^{0.36} \left[ 1 - \left( \frac{u_*}{w_f} \right)^2 \right] \quad (6)$$

The term  $(1 - q_s / q_t)$  was introduced for modeling the cover effect: *the rock surface is covered by particles so that the abrasion due to saltation will be reduced; abrasion will stop when the sediment load reaches to the transport capacity.* The term

$$\left[ 1 - \left( \frac{u_*}{w_f} \right)^2 \right]$$

was for the suspension effect: *when the flow is sufficiently strong ( $u_* \rightarrow w_f$ ), all the sediment particles will be suspended; the saltation is replaced by suspension, and the abrasion is stopped.*

The erosion rate is also inversely related to the sediment mobility factor  $(\tau / \tau_c - 1)$ , often produces results of large erosion rate with  $(\tau / \tau_c - 1) \rightarrow 0$ . This will cause instability when shear stress is a small. To eliminate the problem, the saltation length Equation (3) is also modified to

$$L_s = 8D_s \quad (7)$$

It was found necessary that additional modifications should be made to Equation (6) before it can be applicable to the field problem. In sediment transport study, it is generally considered that sediment starts to become suspended when  $u_* \rightarrow w_f$  but the equations (4) and (5) assume all the sediments are in suspension, and no particles touch the bed. Secondly, abrasion could be present when sediments roll over the bed, so when saltation is weakened and the bed is covered with rolling and sliding bed load materials, the abrasion could still exist but in a reduced magnitude. For these reasons, the suspension and cover factors are modified using an exponent function:  $e^{-C_s(u_*/w_f)^2}$  or  $e^{-C_q(q_s/q_t)}$ . Figure 1 compares the exponent function and that of the original. The sudden stop of the erosion when  $u_* \rightarrow w_f$  or  $q_s \rightarrow q_t$  will be replaced by a gradual variation. These modifications allow soft rock erosion to continue when  $u_* \geq w_f$  or  $q_s \geq q_t$ .

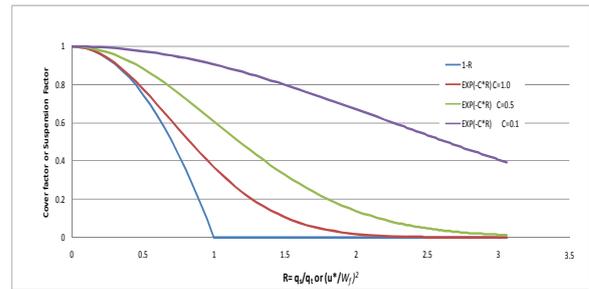


Figure 1 Comparison of the exponent functions to the cover and suspension factors

Considering all these modifications, equation (6) becomes

$$E = C_{\text{coeff}} 0.08 \frac{Y}{k \sigma_T^2} \left[ \left( \frac{\rho}{\rho} - 1 \right) g \right] q_s \left( \frac{\tau}{\tau_c} - 1 \right)^{0.36} e^{-C_q q_s / q_t} e^{-1.5 C_s (u_*/w_f)^2} \quad (8)$$

Equation (8) is applied to simulate the rock bed scouring process, with parameters  $C_s$ ,  $C_q$  and  $C_{coeff}$  to be calibrated for computing the site specific rock erosion rate. It was found that  $C_s = 0.01$  and  $C_q = 0.1$  produce the best erosion pattern over the erodible bed rock, and these two parameters were then used to calibrate  $C_{coeff}$ . Because  $C_{coeff}$  is introduced for site specific calibration, and the true value of the parameter  $Y / \sigma_T^2$  for the site of the JiJi Weir is also unknown, the combined parameter  $C_{Coeff} Y / \sigma_T^2$  was calibrated.

In this study, both suspended load and bed load were computed using the CCHE2D model with six sediment size classes. Computationally determined bed load  $q_s$  and the bed load transport capacity  $q_t$ , determined by the formula of Wu, et al. (2000) for each sediment size were used in Equation (8). The erosion due to each sediment size group was considered. Since the bed rocks at this site are exposed in dry season with little discharge in the channel, it is assumed to have no deposition or static cover in the computation all the time. An *Erodible Rock* type of nodes was created in the computational code which can be eroded by used Equation (8) with no depositions allowed. Since the bed rock is very soft and the sediment load in this River is relatively high, the eroded materials for the bed rock were not included in the bed load.

### 3. Simulation of the Rock Bed Scouring Process Downstream of the JiJi Weir

In the channel downstream of the JiJi Weir, the base rock of the channel bed has been eroded considerably. The scour depth reached as much as 10m. A field inspection indicated that the bed rock was covered by a thin layer of coarse bed materials; the bed rock scour started after this layer of sediment was washed away. Figure 2 shows the scoured bed, the sediment layer cover and the higher point bar. The erosion is mainly concentrated in the main channel, the point bar nearby only has deposition. From the field inspection, it seems that there is no deposition over the bare base rock. The flow and the sediment normally just pass over the base rock bed.

Figure 2 Scoured bed of the channel downstream of



the JiJi Weir.

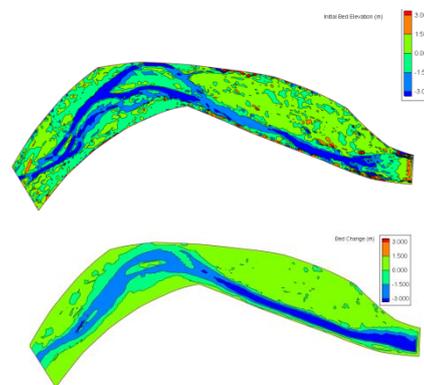


Figure 3 Measured (top) and computed (bottom) rock bed incision and channel deposition pattern downstream of JiJi Weir.

In Figure 3, the measured bed elevation change and computed bed elevation change were compared, with the lower one being the computed and the color bar for both cases being scaled the same. The overall pattern of channel change is similar to what has been observed. The rockbed channel incision pattern agrees with the data better than that of the deposition over the point bar, because only flows with high peak discharges were able to run over the flood plain for a short time.

The measured bed elevation change of the channel downstream of the JiJi Weir was based on 2004 to 2007 DEM bed elevation data. The scouring channel is indicated by the blue contours (Fig. 3), the areas with light green are also eroded somewhat. The deposition occurred only over the big point bar on the right side of the channel. Most deposition is less than 1.5 meters. The bed elevation surface in 2004 was directly used to generate the computation mesh which was then used to simulate the flow, sediment transport and rockbed incision. It was found via this calibration

that  $C_{Coeff} Y / \sigma_T^2 \approx 200,000.0$ . Apparently, the erodibility of the bed rock in the channel downstream of the JiJi Weir is substantially larger than those used in the experiment (Sklar and Dietrich, 2001, 2004).

The general pattern of this simulation result confirmed that the CCHE2D model and the added rockbed incision module can predict the observed flow, sediment transport and rock scouring process. One has to note, however, some uncertainties are involved in the simulation:

- The bedrock is assumed homogeneous
- The sediment load and size compositions of the boundary condition at the JiJi Weir were estimated
- The erosion rate formula, Eq. 8, is used with no additional calibration with original experimental data
- The calibrated erosion rate coefficient would be dependent on the sediment boundary condition. The higher the amount of bed load comes with the flow, the lower the coefficient would be.
- The adaptation length for bedload transport and suspended load adaptation coefficient over normal bed ( $L_n, \alpha_n$ ) and erodible rock ( $L_r, \alpha_r$ ) are assigned differently, according to the nature of the sediment transport. For the normal bed with sediment bed materials:  $L_n = 1000m, \alpha_n = 0.01$ ; for erodible rock bed:  $L_r = 10^8 m, \alpha_r = 0.01$ . Using a very large number for bed load adaptation length over erodible rock bed is based on the observation that bedload passes over the the rock surface without deposition in this study reach.
- The eroded rock materials are assumed to be very fine sediment particles (wash load) and were not added to the bed load.

#### 4. Validation of Rock Bed Erosion Model

The developed rock bed erosion model and calibrated model parameters were tested using the observed flow processes and estimated sediment loads in the period from Jan. 2007 to June 2008. Figure 4 shows the prediction results. The main channel with erodible soft rock was eroded and the point bars and islands have some deposition. The

erosion and deposition over the high flood plain are of less importance.

The flow hydrograph used has been simplified to include only high flow events. The total simulation time was about twelve days and eighteen hours. It was assumed that 90% of the sediment entering the downstream channel was in the form of suspended load, and 10% was bed load. This ratio is the same as that used for the calibration. The simulated erosion over the rock bed is ranging from 1.0 to 2.2 meters with the deeper erosion appearing near the site of the Weir. The deepest erosion (~3.2 m) part is very close to the Weir, and the area is very small. It is possible that this is due to the error of data preparation.

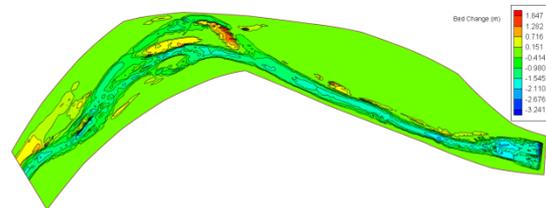
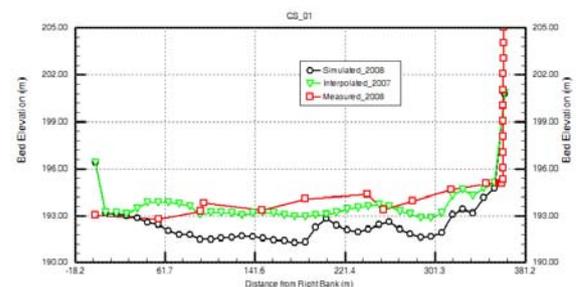
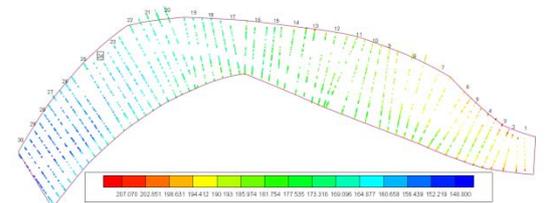


Figure 4 Predicted bed change in the soft rock channel downstream of JiJi Weir.



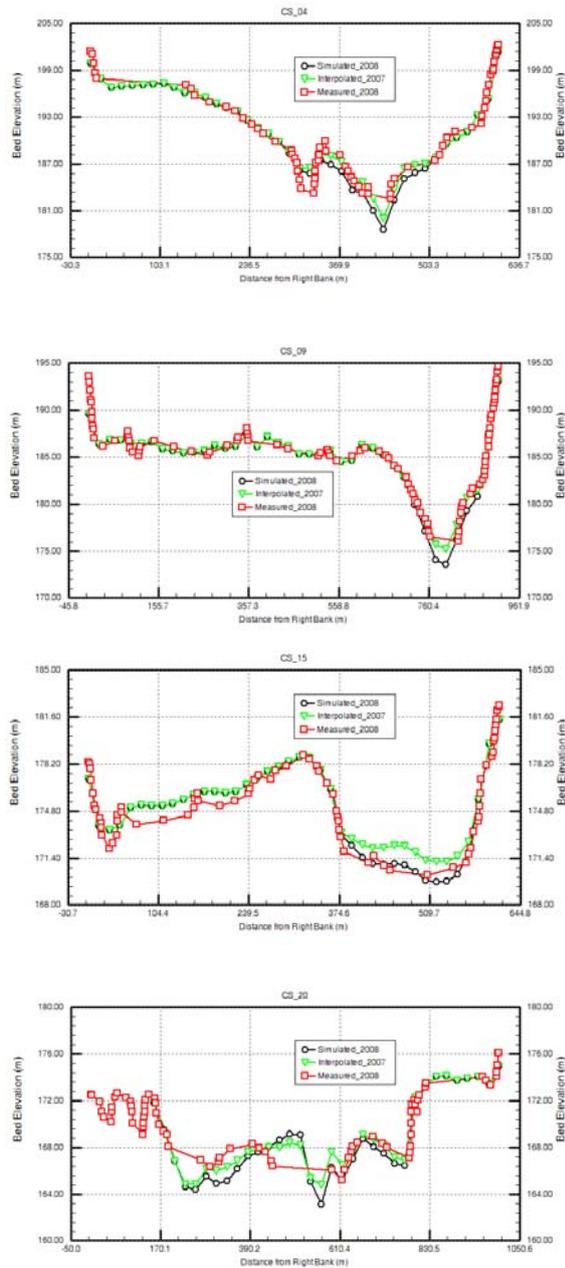


Figure 5 Comparison of predicted and measured soft rock bed elevation is selected channel cross-sections. Locations of cross-sections (top), and several compared cross-sections.

The comparisons of the prediction and measured cross-sections indicate (Fig. 5) that the computed soft rock erosion agreed with measurement reasonably well, except very close to the JiJi Dam (CS 1-3) or far from the Dam (CS 20+). Very close the Dam, the model predicted erosion, but the measurement showed little change; at far downstream, the model predicted some erosion but the measurements indicated some deposition; between these two, the predicted soft rock erosion

is quite consistent with the measurement. This result shows the usefulness and limitation of the model: it gives good prediction at where soft rock erosion occurred; the prediction will not be realistic if deposition occurred because the bed rock model does not include deposition mechanism.

## 5. Conclusions

CCHE2D model has been applied to simulate the bed rock erosion processes downstream of JiJi Weir. The model has been modified to handle the bedrock erosion and representing different erosion control structures. The bed rock erosion formula of Sklar and Dietrich (2001) was modified to be feasible and applicable to the field engineering problem.

The flow processes have been simplified to include only large flow discharges. The general pattern of channel erosion simulated by the developed bed rock erosion model was in agreement with the observation. The validation run shows that the model produced realistic results in the cross-sections with rock erosion.

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## References

- Jia, Y., Wang, S.Y.Y., and Xu, Yichen, 2002, "Validation and application of a 2D model to channels with complex geometry", *International Journal of Computational Engineering Science*, Vol. 3, No. 1 (March 2002), pages 57-71
- Sklar, L.S., Dietrich, W.E., (1998), "River longitudinal profiles and bedrock incision models: Stream power and the influence of sediment supply", in *Rivers over Rock: Fluvial Processes in Bedrock Channels*, Geophysics Monogr. Ser., Vol. 107, edited by K. Thinkler and E.E. Wohl, pp237-260, AGU, Washington, D.C.
- Sklar, L.S., and Dietrich, W.E., (2001), "Sediment and rock strength controls on river incision into bed rock", *Geological Society of America, Geology*, 29(12), p. 1087-1090.

Sklar, L.S., and Dietrich, W.E.(2004), "A mechanistic model for river incision into bedrock by saltating bed load", *Water Resources Research*, Vol. 40, W06301, P:1-21.

Stock, J.D., Montgomery, D.R., Dietrich, W.E., and Sklar, L., 2005, "Field measurements of incision rates following bedrock exposure: Implications for process controls on the long profiles of valleys cut by rivers

and debris flows", *GSA Bulletin*, January/February 2005, V117, No. 11/12, p174-194

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- Wu, W., S.S.Y., Wang, and Y, Jia (2000). "Non-uniform sediment transport in alluvial rivers", *Journal of Hydraulic Research*, Vol. 38(6):427-434.